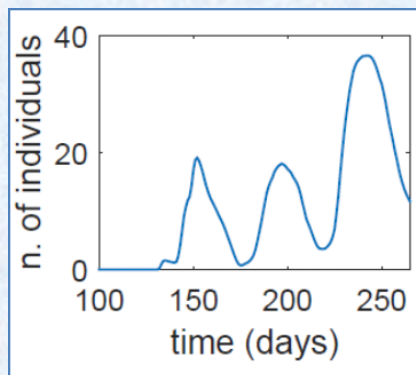
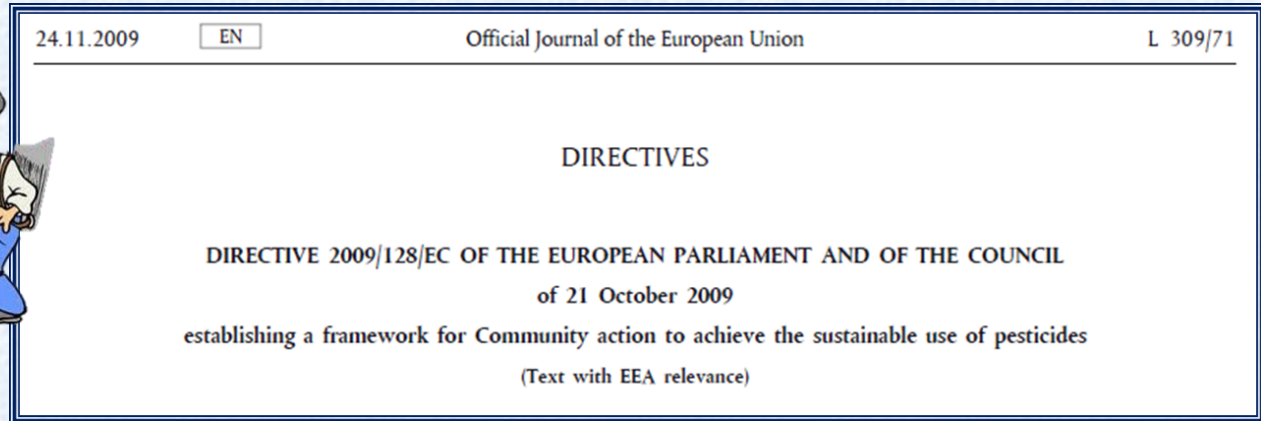


Modelli stocastici nella lotta ai parassiti

Sara Pasquali – CNR IMATI Milano



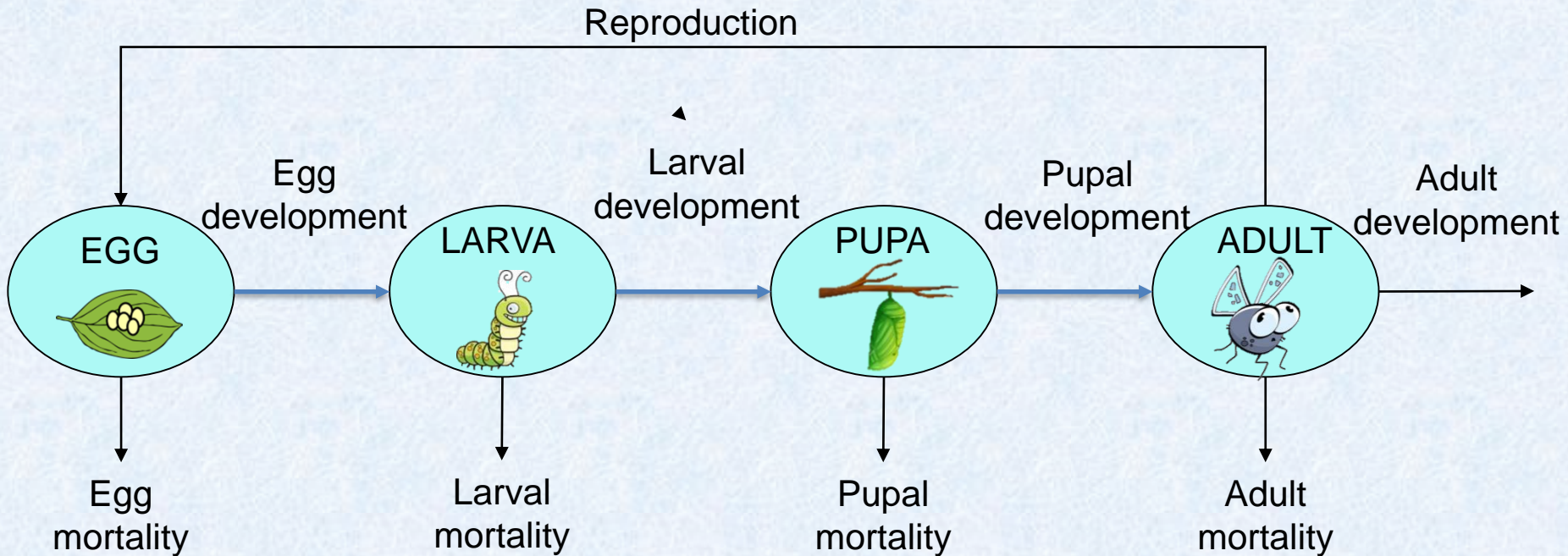
Sustainable pest management



ADVANTAGES

- Reduce environmental impact
- Reduce health impact
- Increase product quality

Stage-structured population



Population dynamics models

$$\frac{\partial \phi^i}{\partial t} + \frac{\partial}{\partial x} \left[v^i(t) \phi^i - \sigma^i \frac{\partial \phi^i}{\partial x} \right] + m^i(t) \phi^i = 0, \quad t > t_0, \quad x \in (0,1)$$

$$\left[v^i(t) \phi^i(t, x) - \sigma^i \frac{\partial \phi^i}{\partial x} \right]_{x=0} = F^i(t)$$

$$\left[-\sigma^i \frac{\partial \phi^i}{\partial x} \right]_{x=1} = 0$$

$$\phi^i(t_0, x) = \hat{\phi}^i(x) \quad i = 1, 2, \dots, s$$

Fokker-Planck
equations

x = physiological age (percentage of development in a stage)

$v^i(t)$ = development rate function in stage i

$m^i(t)$ = mortality rate function in stage i

Population dynamics models

$$\frac{\partial \phi^i}{\partial t} + \frac{\partial}{\partial x} \left[v^i(t) \phi^i - \sigma^i \frac{\partial \phi^i}{\partial x} \right] + m^i(t) \phi^i = 0, \quad t > t_0, \quad x \in (0,1)$$

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$$\phi^i(t_0, x) = \hat{\phi}^i(x) \quad i = 1, 2, \dots, s$$

Fokker-Planck
equations

$\phi^i(t, x) dx$ = number of individuals in stage i with age in $(x, x + dx)$

$N^i(t) = \int_0^1 \phi^i(t, x) dx$ n. of individuals in stage i at time t

$F^1(t) = \int_0^1 b(t) f(x) \phi^s(t, x) dx$ egg production flux

$F^i(t) = v^{i-1}(t) \phi^{i-1}(t, x) \quad i > 1$ flux from a stage to the next

σ^i constant diffusion coefficients $\hat{\phi}^i(x)$ initial distributions

Physiological age: Wiener vs. Gamma

$$\frac{\partial \phi(t, x)}{\partial t} + \frac{\partial}{\partial x} \left[v(t) \phi(t, x) - \sigma \frac{\partial \phi(t, x)}{\partial x} \right] = 0 \quad t > t_0$$

$\phi(t, x)$ denotes the p.d.f. of the physiological age $X_N(t)$ satisfying

$$dX_N(t) = v(t)dt + \sqrt{2\sigma}dw(t) \quad X_N(0) = 0 \quad w(t) \text{ Wiener process}$$

This equation allows regressions in the physiological age.

1

$$dX_G(t) = \tilde{v}(t)dt + dL(t) \quad X_G(0) = 0$$

$L(t)$ homogeneous Gamma process with shape function αt and rate μ

2

$$dX_G(t) = dL(t) \quad X_G(0) = 0$$

$L(t)$ inhomogeneous Gamma process with shape function $\alpha(t)$ and rate μ

Population dynamics (Gamma)

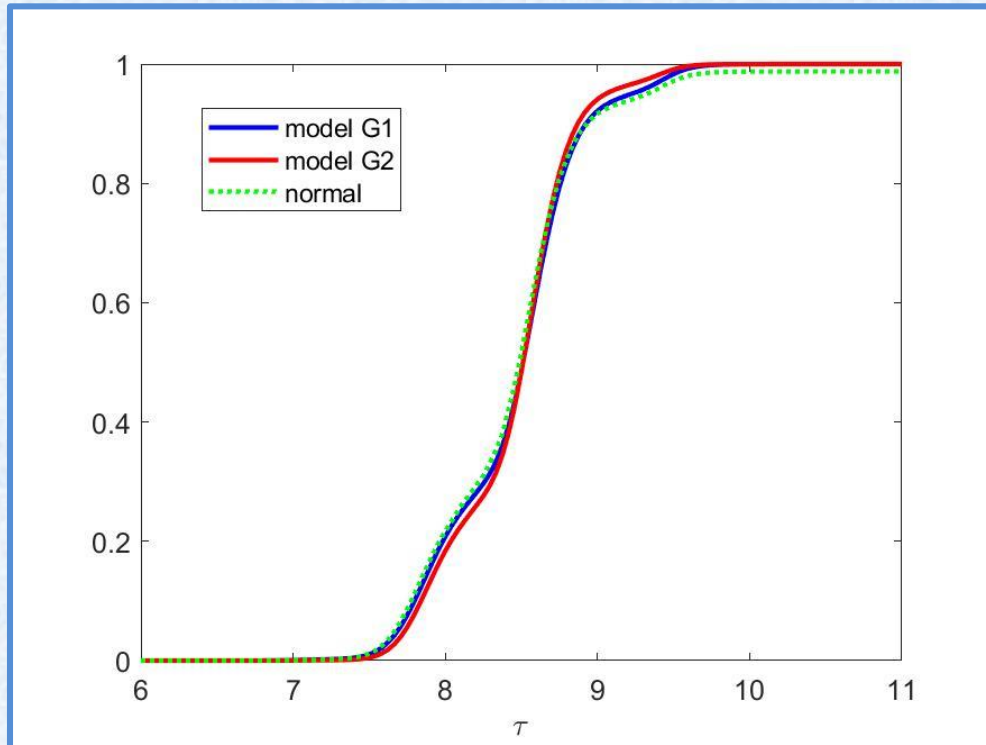
$$E(X_G(t)) = E(X_N(t))$$



$$\tilde{v}(t) = v(t) - \frac{\alpha}{\mu} \quad \mathbf{1}$$

Constraint
on $v(t)$

$$\alpha(t) = \mu \int_0^t v(s) ds \quad \mathbf{2}$$



Cumulative distributions of
the residence time

Population dynamics models

$$\begin{aligned} \frac{\partial \phi^i}{\partial t} + \frac{\partial}{\partial x} \left[v^i(t) \phi^i - \sigma^i \frac{\partial \phi^i}{\partial x} \right] + m^i(t) \phi^i &= 0, \\ \left[v^i(t) \phi^i(t, x) - \sigma^i \frac{\partial \phi^i}{\partial x} \right]_{x=0} &= F^i(t), \quad t > t_0, \quad x \in (0, 1) \\ \left[-\sigma^i \frac{\partial \phi^i}{\partial x} \right]_{x=1} &= 0 \\ \phi^i(t_0, x) &= \hat{\phi}^i(x) \quad i = 1, 2, \dots, s \end{aligned}$$

Buffoni and Pasquali, 2007
Structured population
dynamics: continuous size and
discontinuous stage structure
J. Math. Biol. 54: 555-595

$v^i(t), m^i(t)$ development and mortality rate functions

$b(t)f(x)$ fecundity rate function

Biodemographic
functions

Good estimate of
biodemographic functions



Reliable population
dynamics model

Biodemographic functions

$v^i(t), m^i(t)$ development and mortality rate functions

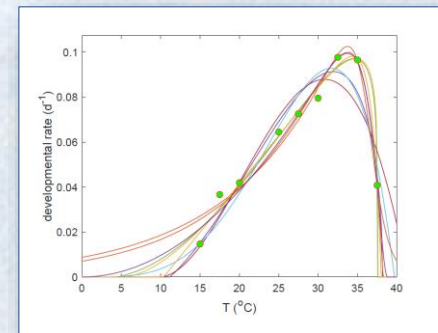
$b(t)f(x)$ fecundity rate function

Biodemographic functions

Data on the biology of the species available



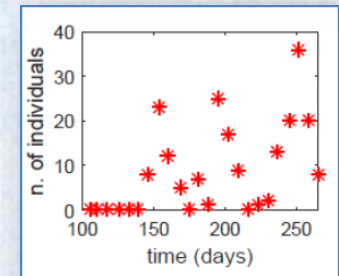
Least square method



Data on the biology of the species NOT available



Statistical estimation method based on population dynamics data



In the following we focus on the estimation of the mortality using population dynamics data

Mortality estimate

CASE 1

$m^i(t)$ of known functional form

least square method

Bayesian method

Gilioli, Pasquali, Marchesini, 2016
A modelling framework for pest population dynamics and management: An application to the grape berry moth - Ecol. Model. 320: 348-357

Lanzarone, Pasquali, Gilioli, Marchesini, 2017
A Bayesian estimation approach for the mortality in a stage-structured demographic model
J. Math. Biol. 75: 759-779

Mortality estimate

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CASE 2

$$m^i(t) = \sum_{j=1}^{n_i} \alpha_{ij} \xi_{ij}(t)$$

$\xi_{ij}(t)$ suitable basis (ex. splines)

α_{ij} parameters to be estimated

Wood, 2001
Partially specified ecological models
Ecol. Monograph. 71(1): 1-25

Pasquali and Soresina, 2022
Mortality estimate driven by population abundance field data in a stage-structured demographic model. The case of *Lobesia botrana*. *Ecological Modelling*, 464, 109842.

Mortality estimate

$$m^i(t) = \sum_{j=1}^{n_i} p_{ij} \xi_{ij}(t)$$

Wood, 2001
Partially specified ecological models
Ecol. Monograph. 71(1): 1-25

$\xi_{ij}(t)$ cubic spline basis

p_{ij} parameters to be estimated

u measurement error -- real abundance in $[Y, Y + u]$

Objective: minimize $\sum_i \max \{0; (Y^i - N^i(t)) w_i (Y^i + u^i - N^i(t))\}$

w_i weight Y^i observation $N^i(t)$ simulated abundance

$\mathbf{p} = (p_{ij})_{i=1, \dots, s; j=1, \dots, n_i}$ vector of parameters

\mathbf{Y} vector of observations

\mathbf{N} vector of simulated abundances

Mortality estimate

- Starting from an initial value for \mathbf{p} , we solve the system of PDE to obtain the simulated abundances at the observation times
- We consider slight changes in the parameter to approximate the Jacobian of N

$$J_{ij} = \frac{N^i(\mathbf{p} + \delta_j \mathbf{e}_j) - N^i(\mathbf{p} - \delta_j \mathbf{e}_j)}{2\delta_j}$$

δ_j small increments ; \mathbf{e}_j vectors of the canonical basis

- we construct a quadratic model to approximate the fitting objective

$$q(\mathbf{p}) = \sum_i \max \left\{ 0; \left(Y^i - \sum_h j_{ih} p_h \right) w_i \left(Y^i + u^i - \sum_h j_{ih} p_h \right) \right\}$$

- we minimize $q(\mathbf{p})$ and obtain a parameter \mathbf{p} used to repeat the procedure until convergence.

At the end of the process we obtain the optimal parameter $\bar{\mathbf{p}}$

Confidence bands

For large samples, the estimator $\hat{\mathbf{p}} = \operatorname{argmin}_{\mathbf{p}} q(\mathbf{p})$ has approximately a multivariate normal distribution with mean $\bar{\mathbf{p}}$ and covariance matrix

$$C_J(\hat{\mathbf{p}}) = \frac{q(\hat{\mathbf{p}})}{d - n_p} (\mathbf{J}^T \mathbf{W} \mathbf{J})^{-1}$$

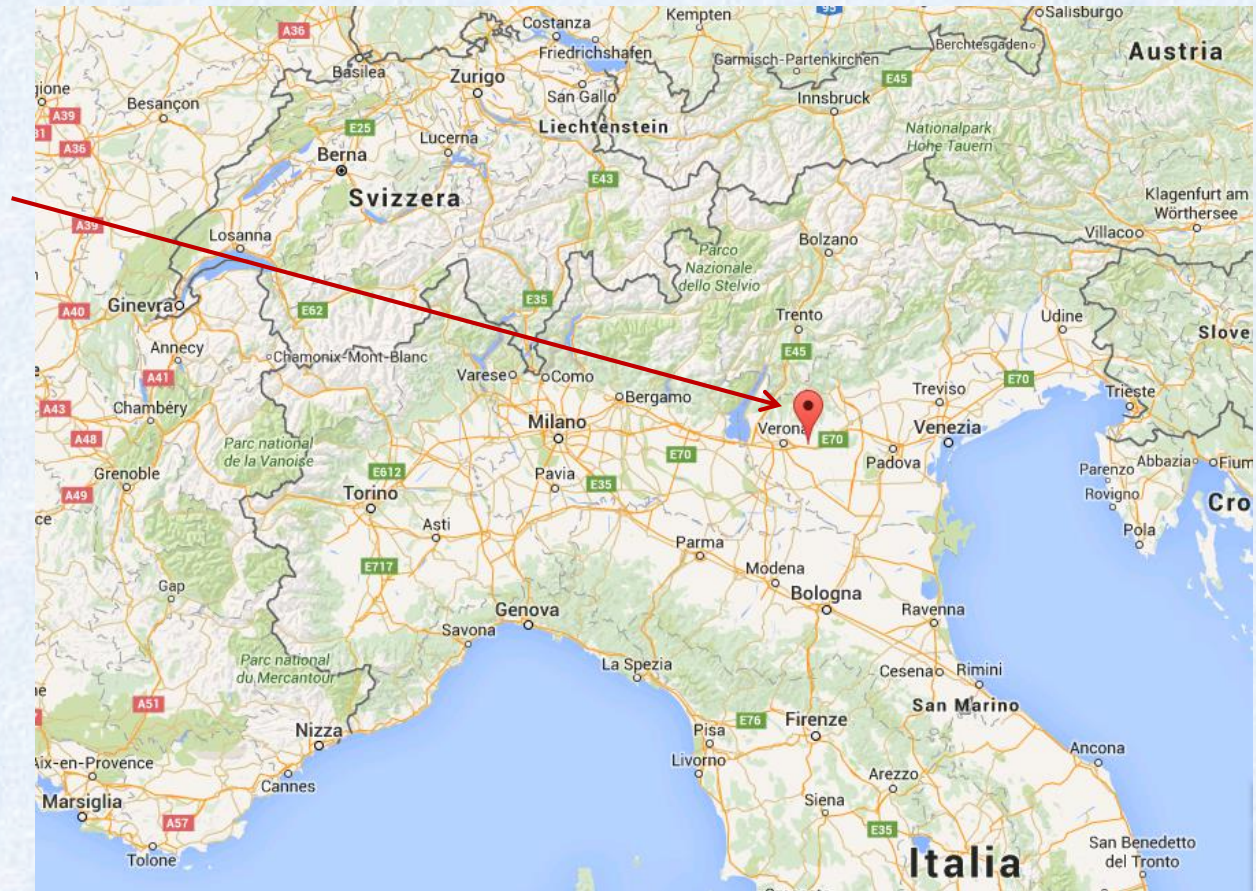
d = n. of observations and n_p = n. of parameters

- We draw a certain number of parameter values from the multivariate normal distribution corresponding to different mortality functions
- With a MATLAB routine we obtain the confidence bands for mortality
- The different mortalities produce different dynamics from which we obtain confidence bands for the dynamics

Confidence bands give a measure of the uncertainty in the estimates.

Case study: the grape berry moth

Estimate the mortality in order to fit the abundance data collected in a vineyard in Colognola ai Colli (Italy) for three years (2008, 2009 and 2011)



Data collected for the cultivar Garganega, from April to September (grape harvest).
Immatures: on a sample of 100 bunches of grapes.
Adults: pheromone traps.

Data on grape berry moth

INPUT DATA FOR MODEL SIMULATION

- Temperatures recorded by a meteorological station placed nearby the vineyard



- N. of adults catches per week until larvae of the first generation are detected



FURTHER DATA FOR MORTALITY ESTIMATION

Weekly data for



• eggs



• larvae



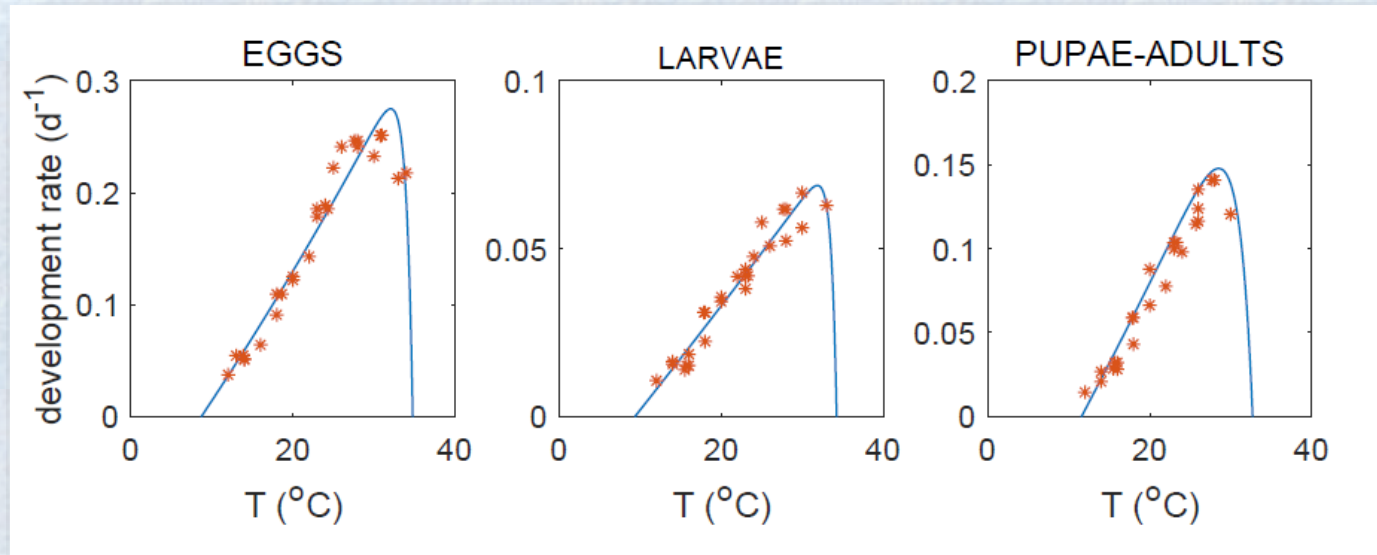
• pupae



• adults

L. Botrana biodemographic functions

Development:
Lactin functions

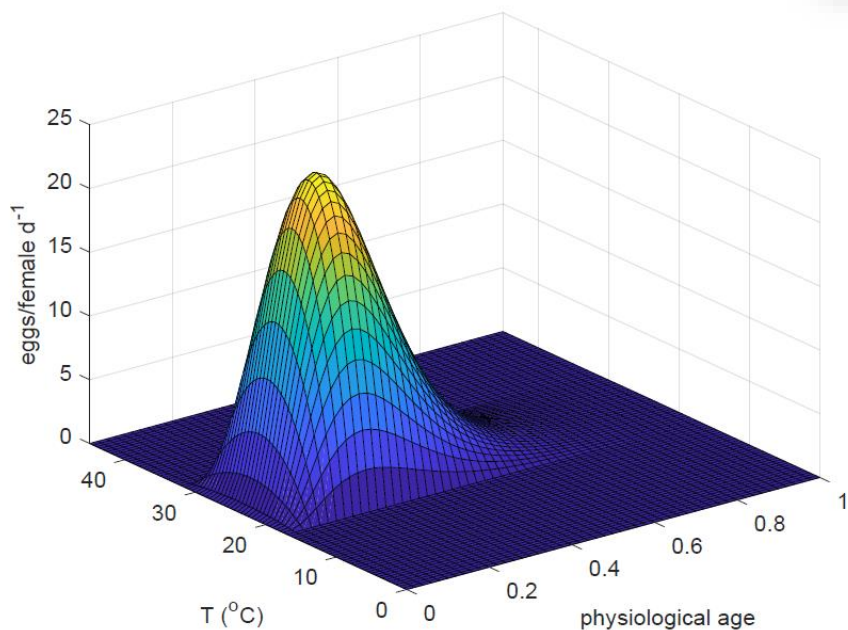
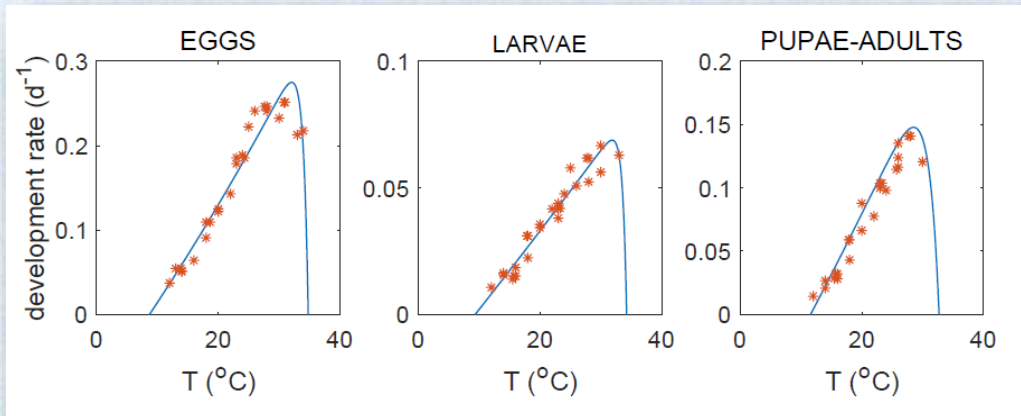


Same development rate function for pupae and adults

Gutierrez, Ponti, Cooper, Gilioli, Baumgärtner, Duso, 2012
Prospective analysis of the invasive potential of the European grapevine moth *Lobesia botrana* (De & Schiff.) in California. *Agr. Forest Entomol.* 14, 225–238.

L. Botrana biodemographic functions

Development:
Lactin functions



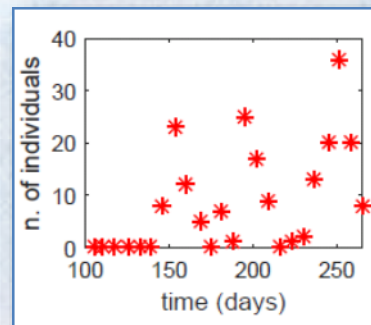
Fecundity function

$$ax^b e^{-cx} \left[1 - \left(\frac{T - T_L - T_0}{T_0} \right)^2 \right]$$

L. Botrana mortality rate function

$m^i(T)$ mortality rate function

?? No literature data



Data on population dynamics to estimate mortality rate function

$$m^i(t) = \sum_{j=1}^5 p_{ij} \xi_{ij}(t)$$

$\xi_{ij}(t)$ cubic splines built on the nodes [0,20,40]

Objective: minimize $\sum_i \max \{ 0; (Y^i - N^i(t)) w_i (Y^i + u^i - N^i(t)) \}$

Weights for larvae >> weights for the other stages

- Grape berry moth particularly dangerous in larval stage
- Measurements of larvae more reliable than for the other stages

u^i 10% for larvae, 50% for eggs and pupae

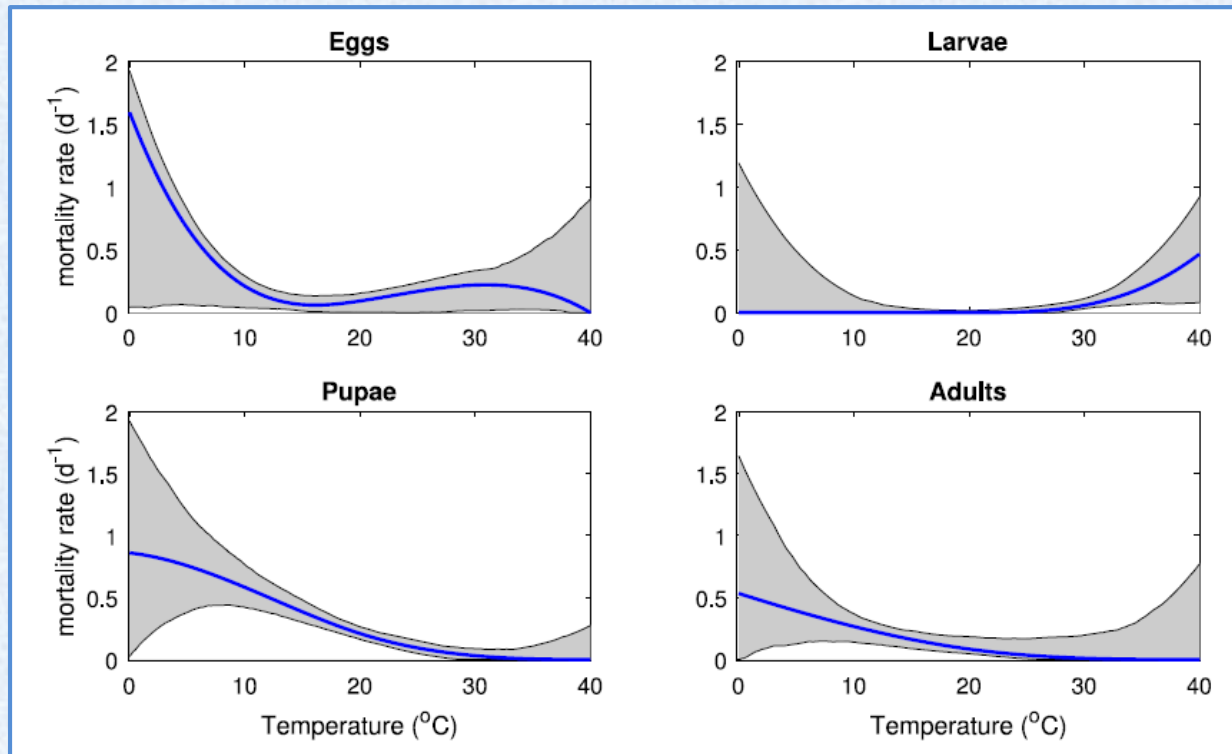
L. Botrana: mortality

Use data for 3 years:

2008

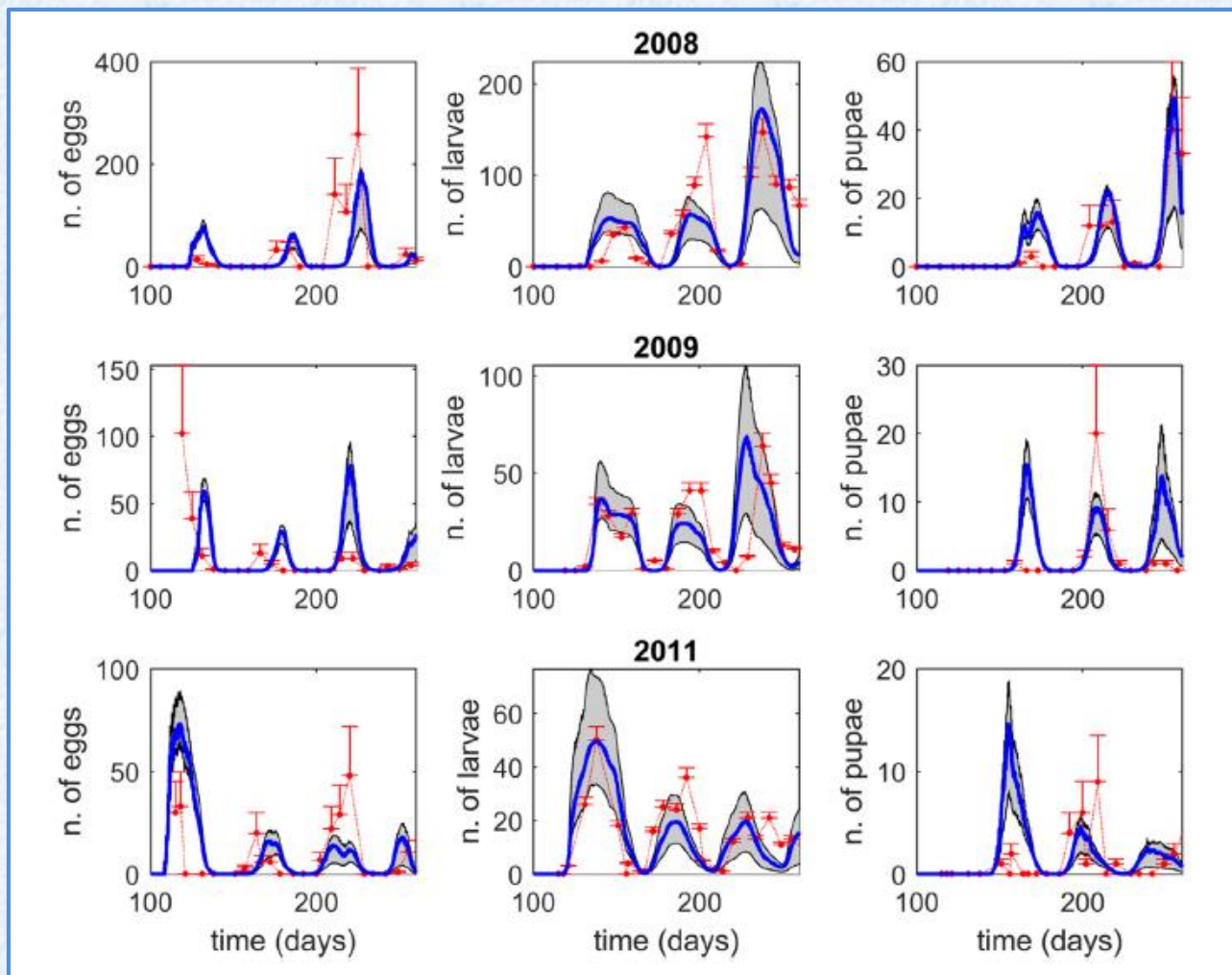
2009

2011



95% confidence bands for mortality: obtained drawing 500 values of the parameter vector from its multivariate normal distribution corresponding to 500 mortalities for each stage

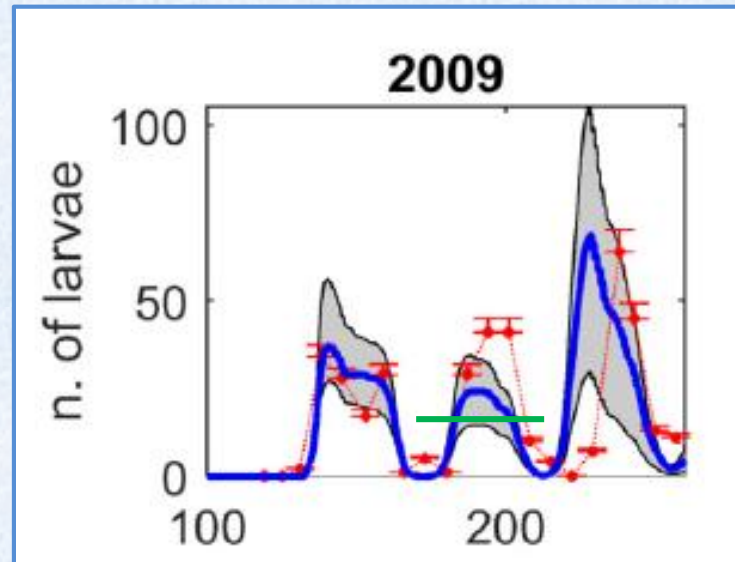
L. Botrana: dynamics



HIGHER WEIGHT FOR LARVAE

Pest control

Population dynamics models can be used to forecast the dynamics and control if alert thresholds are crossed.



Larvae of 2nd generation: most damaging stage

- Red: collected data
- Blue: simulated dynamics
- Green: control threshold for treatment in 2nd generation

Web service



DINAMICA DI POPOLAZIONI INVASIVE

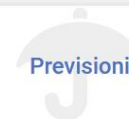
Insetti



Fenologico



Demografico



Previsioni



Riferimenti

Sara Pasquali
Cinzia Soresina
Maria Teresa Artese

IMATI - Sezione di Milano

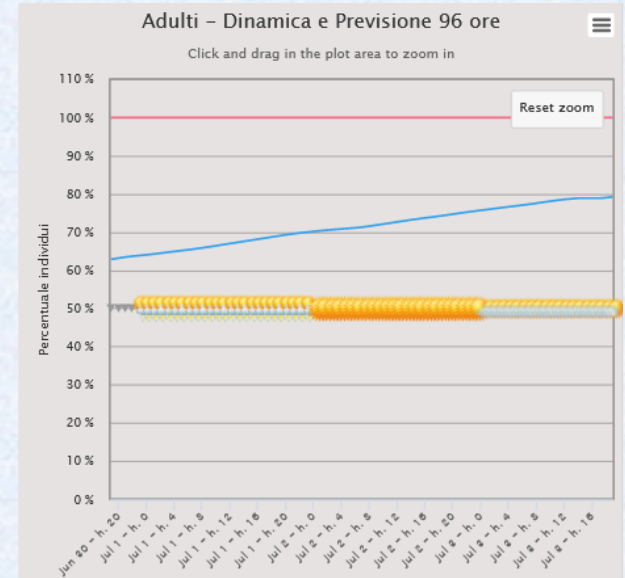
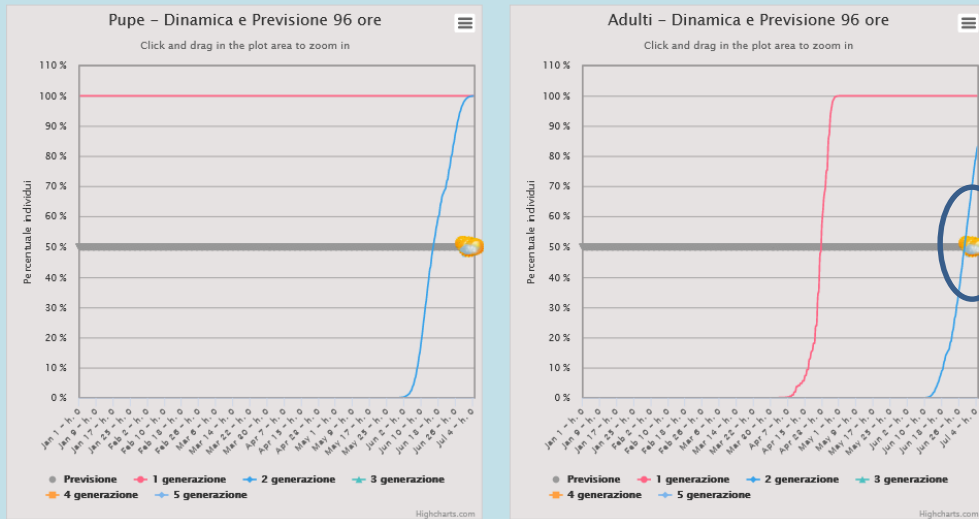
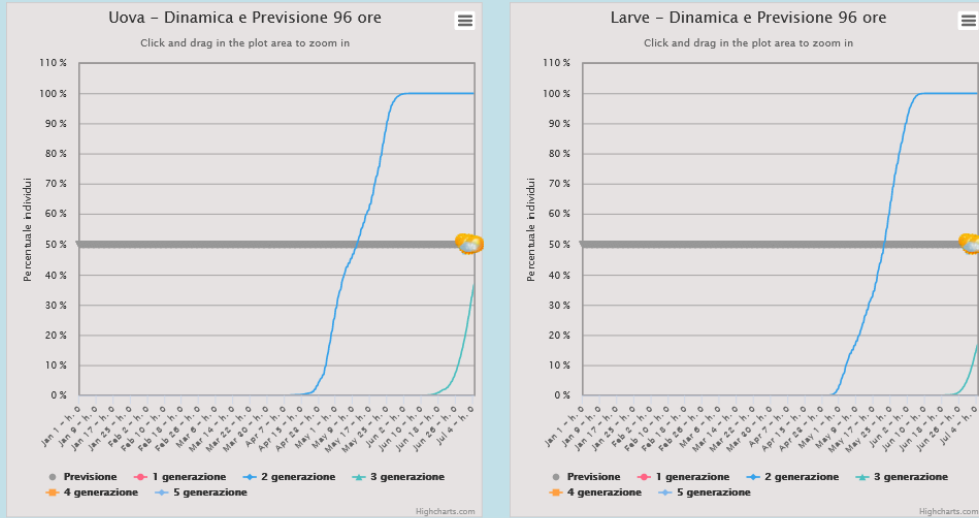
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20133 Milano
Italy

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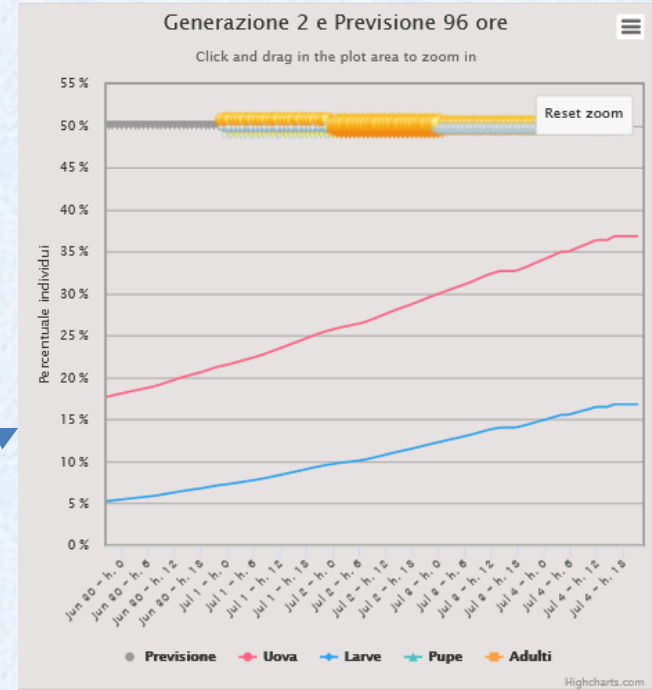
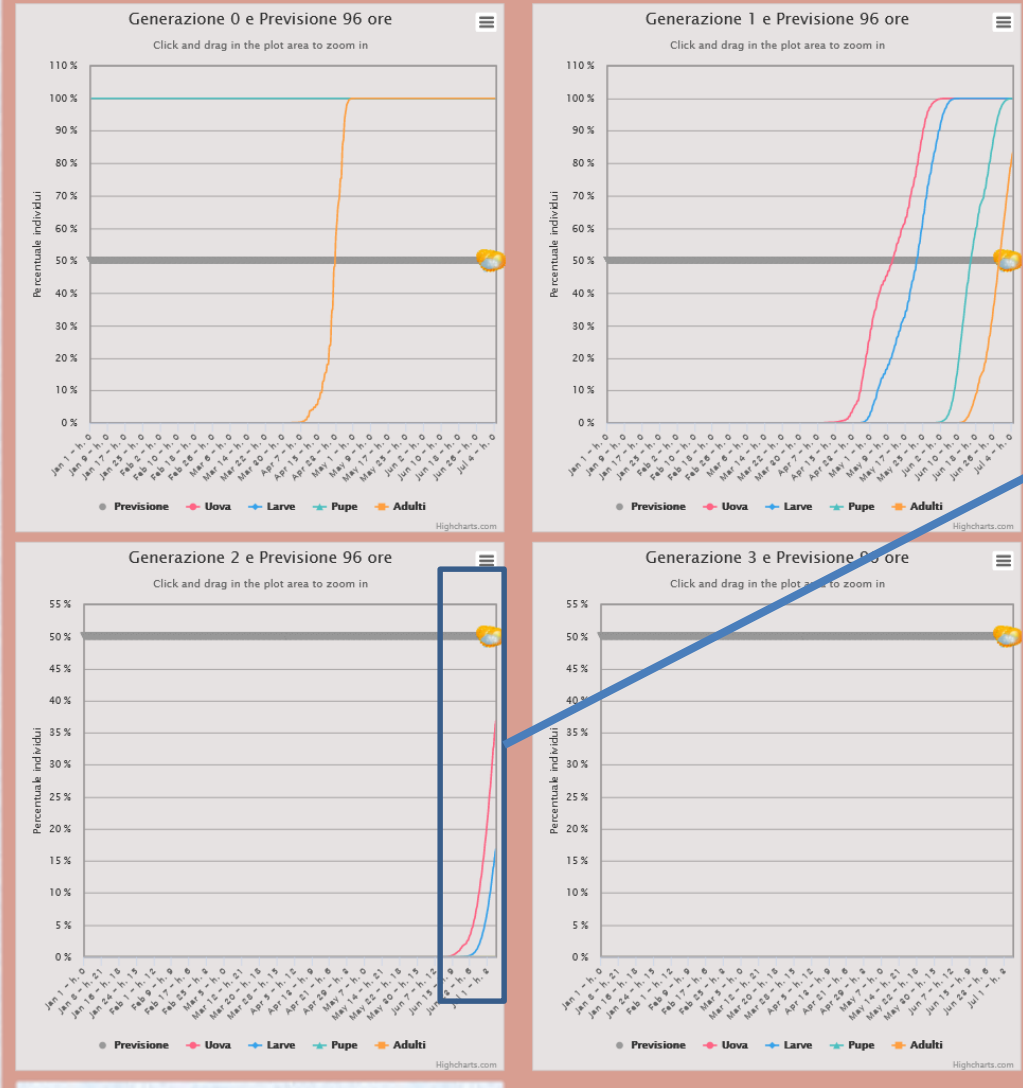
Web service

GRAFICI PER STADI



Web service

GRAFICI PER GENERAZIONI



GRAZIE

